Can string theory cure heart rhythm disorders? (*)

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What on earth have a beating heart and a cosmic string in common?

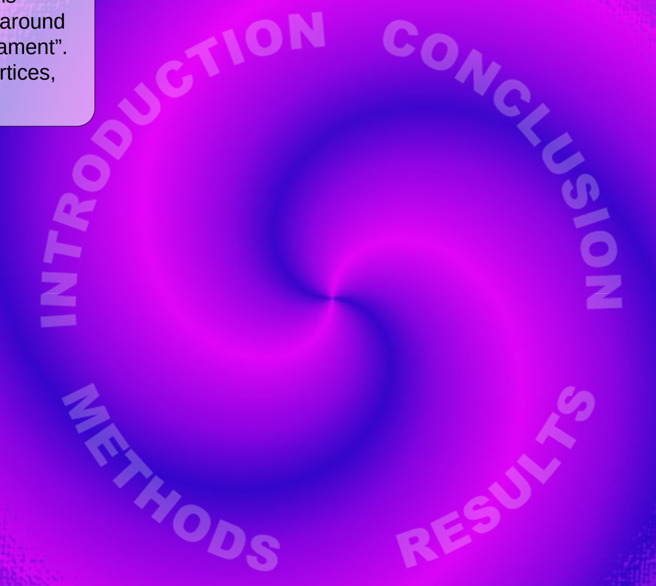
- Some types of heart rhythm disorders: rotating spirals on the heart's surface.
- Rotation axis = "filament"
- Filament dynamics closely resembles cosmic strings.

What do we do?

- Start from generic reaction-diffusion equations describing electric activation in the heart muscle
- Borrow mathematical methods from geometry, string theory and general relativity
- Look for solutions to the equations
- Translate findings to a cardiac context

What are you supposed to do ?

Follow the spiral counterclockwise; let the mathematical treatment lead you from cell physiology over geometric considerations to general relativity and string theory, and all the way back to medical implications…



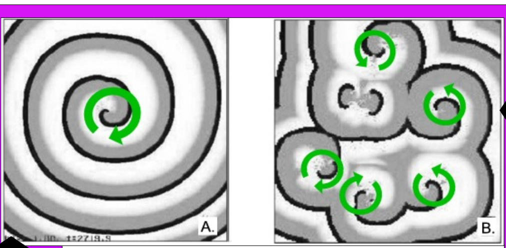
Bushfire, zebra stripes, the Mexican wave

- … and cardiac tissue are all examples of excitable media.
- Point stimulus can trigger activation waves
- Recovery time after each excitation
- Underlying reaction-diffusion equation:

$$\partial_t \mathbf{u} = \partial_i\left(D^{ij}\partial_j \mathbf{u}\right) + \mathbf{\Phi}(\mathbf{u}) \qquad (1)$$

Hurricane in the heart

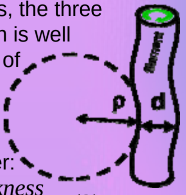
- Depolarization waves are initiated by nerve stimulation.
- In healthy state, near-plane waves propagate, and induce homogeneous contraction of the muscle: one trigger -> one wave -> one contraction
- Some people have tachycardia (100-200 beats/min.): the wave gets trapped in a loop (see Mexican wave).
- One wave -> many unsynchronized contractions
- Isopotential maps consist of spirals that rotate around their tips; the line connecting the tips in 3D: "filament".
- Rotor filaments are similar to hydrodynamic vortices, tornado cores and the eye of a hurricane.



Cardiac tachycardia (A) versus fibrillation (B). Black lines: depolarization fronts, grey zones: tissue in recovery. Adapted from [1].

3D = 2D + 1D

- For weakly bent filaments, the three dimensional wave pattern is well approximated by a stack of 2D spiral waves.
- Mathematics: gradient expansion with expansion parameter:



$$\lambda = \frac{d}{\rho} = \frac{filament\ thickness}{radius\ of\ curvature} \qquad (2)$$

- Inspired by cosmic strings [2])

Anisotropy = curvature

- Myocardium is highly anisotropic due to muscle fibres and cleavage planes.
- Anisotropy affects filament motion.
- Redefine distance ds using a metric tensor based on tissue diffusion properties [3]:

$$ds^2 = \sum_{i=1}^{3}\sum_{j=1}^{3} g_{ij}(\mathbf{x})\,dx^i dx^j,\quad g_{ij} = \left(D^{-1}\right)_{ij} \qquad (3)$$

(Generalization of Pythagoras theorem)

- Filament twisting and bending is studied in the resulting curved space.

Equation of motion

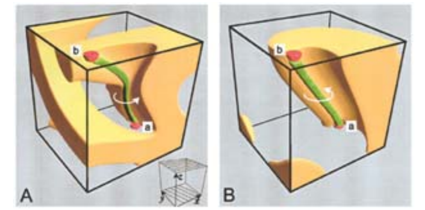
- We analytically derived [4] the equation of motion for filaments in generic anisotropic media.
- They move along normal and binormal vector:

$$\dot{\mathbf{X}} = \gamma_1\frac{\mathbf{N}}{\rho} + \gamma_2\frac{\mathbf{B}}{\rho} + \mathcal{O}\left(\lambda^2\right) \qquad (4)$$

- Physical meaning: γ_1 = string tension.
- $\gamma_{1,2}$ can be evaluated numerically for a given heart model (1).

Statics: geodesic solution

- From (5): no motion $\dot{\mathbf{X}} \to 0$ if filament curvature $1/\rho \to 0$
- In the space (3), this corresponds to a geodesic (= curve of minimal length connecting two given points [3,4]).
- If the heart had been isotropic: $\Rightarrow$ solution = straight line
- Intuitively: a filament with string tension $\gamma_1 > 0$ tends to minimize its length. [5]



With anisotropy [3] Without anisotropy [3]

Realizations

- We have obtained equations of motion for both rotor filaments and wave fronts.
- We take into account arbitrary tissue anisotropy, dispersion effects, bidomain models, parameter gradients and twist.
- The results are presented in terms of geometric invariants such as filament curvature and the tissue's Riemann tensor.

Computational benefits

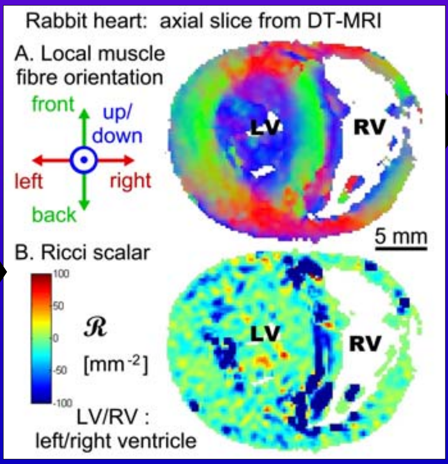
- Current computer simulations solve (1) by forward integration, on a 3D grid with 5-100 variables.
- Simulations of the filament alone, based on (4) and its second-order terms provide a fast alternative to asses time evolution of tachycardia pathologies.

Outlook

- The state of ventricular fibrillation (leading to cardiac arrest) comes with multiple filaments, whose interactions need to be studied.
- Our formalism doesn't include the coupling with contraction mechanics yet.

The Riemann tensor of the heart

- In curved space, it is important to work with coordinate invariant or tensor quantities.
- Curvature of space itself is contained in the Riemann curvature tensor R_{ijkl}.
- The trace of this tensor is a scalar invariant of the space, denoted $\mathcal{R}$ (= Ricci scalar).
- Classification of spaces: $\mathcal{R}$=0: flat space, $\mathcal{R}$>0: sphere-like, $\mathcal{R}$<0: saddle-like
- Using anatomical data (e.g. MRI scans), we were the first to calculate R_{ijkl} and $\mathcal{R}$ for a given heart.



Corrected angular velocity

- Spiral waves rotate slower or faster due to curvature effects 1/ρ (filament bending) and R_{ijkl} (curvature of surrounding space, i.e. tissue anisotropy).

$$\omega = \omega_0 + p_1\rho^{-2} + p_2\mathcal{R} + p_3 R_{1212} \qquad (5)$$

- The constant coefficients $p_{1,2,3}$ can be predicted numerically for a given electrophysiological heart model (1).

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References:

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(*): Not yet. But we do our very best to gain better insights into the matter…